Introduction to Hydrology (Geog 3511) - Fall 2002
Steps to solving problems that involve Darcy's Law

Introduction

Darcy's Law is the basic equation used to solve groundwater flow problems in both the saturated and unsaturated zones. Darcy's Law is pretty easy to solve, but it is more difficult to interpret the results because they depend on a series of sign convention choices. Listed below is a series of steps that should help you to solve and interpret problems that involve Darcy's Law.

Step 1 - State your sign convention for measurements along the vertical axis

The vertical sign convention is arbitrary, but the sign convention that you choose determines the equations that you use and how you interpret your results.

Convention A: values increase as you increase in elevation (z)

By choosing this sign convention, the equation for total hydraulic head is

$$h = \psi + h_z$$

where ψ is the pressure head and h_z is the elevation head measured in the z direction.

Convention B: values increase as you decrease in elevation (z')

By choosing this sign convention, the equation for total hydraulic head is

$$h = \psi - h_{z'}$$

where ψ is the pressure head and $h_{z'}$ is the elevation head measured in the z' direction.

Step 2 - State your sign convention for measurements along the direction of flow

Darcy's Law can be used to calculate flow in any direction, but usually we want to calculate flow in a particular direction (vertical, horizontal, or otherwise). Similar to the vertical sign convention, we need to adopt a sign convention for measurements made along the direction that we will calculate flow along. Once again, the sign convention is arbitrary. However, you do need to state a sign convention, so that you can interpret your calculated discharge.

If the freedom of being allowed to arbitrarily choose a sign convention makes you nervous, guess which way the flow will occur and choose that as your sign convention. If you end up guessing the wrong direction, you will calculate a negative discharge.

If you are interested in flow in the vertical direction, you have already chosen your sign convention for the direction of flow in Step 1.

Step 3 - Sketch the problem

Sketch out the problem, making sure the measurements specified in the problem statement do not violate your sign conventions.

Step 4 - State your convention for which way changes are to be measured

The change operator (Δ) has to be applied in the same way throughout the problem. This means if you choose to use $\Delta h = h_2 - h_1$, then you must also use $\Delta l = l_2 - l_1$ (not $\Delta l = l_1 - l_2$). This is particularly important when a problem statement gives you a distance between measurements instead of the location of the two measurements. Depending on the horizontal sign convention, the distance between two measurements (l) could be interpreted as $\Delta l = l_2 - l_1$ or $\Delta l = l_1 - l_2$. Watch out, because only one of the interpretations will be correct!

I will use the convention that the change in a variable is determined by subtracting the second measurement from the first, i.e. $\Delta h = h_2 - h_1$ and $\Delta l = l_2 - l_1$.

Step 5 - Write Darcy's Law

The general form of Darcy's Law is

$$q_l = -K\frac{dh}{dl}$$

where q_x is the specific discharge measured in the l direction, K is the hydraulic conductivity, and $\frac{dh}{dl}$ is the hydraulic gradient in the l direction. Note that the l-direction can be any arbitrary direction - it does not have to be in the horizontal or vertical direction because the general form of Darcy's law is valid in any spatial direction. Normally we estimate the hydraulic gradient by using measurements at two locations (1 and 2) and we write Darcy's Law as

$$q_l = -K\frac{\Delta h}{\Delta l}$$

Using our convention for how changes are measured, we can then write

$$q_l = -K\frac{h_2 - h_1}{l_2 - l_1}$$

Flow in the vertical direction

For flow in the vertical direction, we usually replace the general l-direction with a more specific z-direction. Darcy's Law, written for flow in the vertical (z) direction, is given by

$$q_z = -K\frac{h_2 - h_1}{z_2 - z_1}$$

Flow in the horizontal direction

For flow in the horizontal direction, we usually replace the general l-direction with a more specific x-direction. Darcy's Law, written for flow in the horizontal (x) direction, is given by

$$q_x = -K\frac{h_2 - h_1}{x_2 - x_1}$$

Step 6 - Pick out two measurement locations from your sketch

In order to use the vertical or horizontal versions of Darcy's Law, we need to pick out two measurement location where we know (or can calculate) both the hydraulic head and the location along the axis that are interested in calculating specific discharge. If total hydraulic head is not given in the problem statement, it is often useful to use the fact that the pressure head is zero (by definition) at the water table.

Step 7 - Calculate hydraulic head at each of the two measurement locations

Calculate the hydraulic head at each point using the equations given in Step 1. Be careful to use the correct equation - it depends on what you chose for the vertical sign convention!

Step 8 - Plug-n-chug

Solve for specific discharge by plugging the hydraulic head and location measurements into Darcy's Law.

Step 9 - Interpret your result

The specific discharge found in Step 8 is measured parallel to the positive axes of the coordinate system you set up in Step 2. If the calculated specific discharge is negative, flow is taking place in the opposite direction. Remember that according to Darcy's Law, water always flows from high hydraulic head to low hydraulic head.