Manning’s Equation

Manning’s equation is commonly used to the mean channel velocity of a stream. Manning’s equation can be written as

\[ U = \frac{k}{n} (R_h)^{2/3} (S)^{1/2} \]

where \( U \) is the discharge velocity (in m/s), \( k \) is the unit conversion factor, \( n \) is the Manning’s n coefficient, \( R_h \) is the hydraulic radius (in meters), and \( S \) is the slope (in meter/meter).

The conversion factor, \( k \), is used because the equation is dimensionally inhomogeneous. The value of \( k \) depends on the choice of units used for \( R_h \) and \( U \). Because we are using standard metric units (meters for \( R_h \) and meters/sec for \( U \)) the value of \( k \) is conveniently 1.0. If you consider the units of \( k \), you would use \( k = 1.0 \text{ m}^{1/3} \text{ s}^{-1} \).

Manning’s \( n \) is a dimensionless number that characterizes the resistance of the channel to flow. Higher values of \( n \) indicate greater resistance. Dingman presents a table of typical Manning’s \( n \) values (Table 9-6). Also there is a USGS website that lists the \( n \) values of actual rivers. http://wwwrcamnl.wr.usgs.gov/sws/fieldmethods/Indirects/nvalues/

The hydraulic radius, \( R_h \), is the cross-sectional area of the stream divided by the wetted perimeter

\[ R_h = \frac{A_{\text{stream}}}{P} \]

River channels are commonly much wider than they are deep. Because of this, we can approximate the wetted perimeter of the channel, \( P \), by the width of the channel.

\[ R_h = \frac{A_{\text{stream}}}{P} \approx \frac{d_{\text{stream}} w_{\text{stream}}}{w_{\text{stream}}} \approx \frac{d_{\text{stream}}}{} \]

Because area has units of square meters and the perimeter has units of meters, the hydraulic radius also has units of meters.

Effect of channel bottom

Bottom roughness of a stream has a large effect on the velocity of water in the channel. Lets solve Mannings equation for mean channel velocity for a smooth sand bed (\( n=0.025 \)) and a very coarse weedy bed (\( n=0.075 \)).

Q1) Based on Manning’s equation, what effect will an increased slope have on the depth of water? (Note: do not use numbers to answer this question) (4 points)
We start with Manning’s equation

\[ U = \frac{k}{n} \left( R_h \right)^{2/3} \left( S \right)^{1/2} \]

Assuming that the channel is rectangular and that the width is much greater than depth

\[ R_h = \frac{A_{\text{stream}}}{P} \approx \frac{d_{\text{stream}} w_{\text{stream}}}{w_{\text{stream}}} = d_{\text{stream}} \]

\[ U = \frac{k}{n} \left( d_{\text{stream}} \right)^{2/3} \left( S \right)^{1/2} \]

Rearranging to solve for depth

\[ d_{\text{stream}} = \left( \frac{Un}{k} \right)^{3/2} \left( S^{-1/2} \right)^{3/2} = \left( \frac{Un}{k} \right)^{3/2} S^{-3/4} = \left( \frac{Un}{k} \right)^{3/2} \frac{1}{S^{3/4}} \]

Because slope has a negative exponent, depth decreases as slope increases.

Use the following table to answer questions 2 through 5:

<table>
<thead>
<tr>
<th>Channel 1</th>
<th>Channel 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>cross-sectional area</td>
<td>12.0 m²</td>
</tr>
<tr>
<td>average stream depth</td>
<td>1.5 m</td>
</tr>
<tr>
<td>channel bottom</td>
<td>smooth sand</td>
</tr>
<tr>
<td>slope</td>
<td>0.0006 m⁻¹</td>
</tr>
</tbody>
</table>

Q2) What is the hydraulic radius? (4 points)
If you only assumed that the channel is rectangular

\[ w = \frac{A_{\text{stream}}}{l} = \frac{12.0 \text{ m}^2}{1.5 \text{ m}} = 8.0 \text{ m} \]

\[ R_h = \frac{A_{\text{stream}}}{P} = \frac{A_{\text{stream}}}{w+2d} = \frac{12.0 \text{ m}^2}{8.0 \text{ m} + 2(1.5 \text{ m})} = 1.09 \text{ m} \]

If you also assumed that the width is much greater than the depth. (Note: it is preferrable not to assume this, since you can calculate the hydraulic radius without this simplification)

\[ R_h = \frac{A_{\text{stream}}}{P} \approx d_{\text{stream}} = 1.5 \text{ m} \]

Q3) What is mean stream velocity for the channel with a smooth sandy bed? (4 points)

\[ U = \frac{k}{n} \left( R_h \right)^{2/3} \left( S \right)^{1/2} = \frac{1.0 \text{ m}^{1/3} \text{ s}^{-1}}{0.025} \left( 1.09 \text{ m} \right)^{2/3} \left( 0.0006 \text{ m} \right)^{1/2} = 1.038 \text{ m} / \text{s} \]

Q4) What is mean stream velocity for the channel with a weedy bed? (4 points)

\[ U = \frac{k}{n} \left( R_h \right)^{2/3} \left( S \right)^{1/2} = \frac{1.0 \text{ m}^{1/3} \text{ s}^{-1}}{0.075} \left( 1.09 \text{ m} \right)^{2/3} \left( 0.0006 \text{ m} \right)^{1/2} = 0.346 \text{ m} / \text{s} \]

Q5) What is the ratio of \( U_{\text{sand}} \) to \( U_{\text{weeds}} \)? (4 points)

\[ \frac{U_{\text{sand}}}{U_{\text{weeds}}} = \frac{1.038 \text{ m} / \text{s}}{0.346 \text{ m} / \text{s}} = 3.0 \]

Effect of slope

Q6) Using the approximation that the channel width is much greater than the depth, rearrange Manning’s equation to solve for stream depth. Based on the equation, what effect will an increased slope have on the depth of water? (Note: do not use numbers to answer this question) (4 points)

Start with Manning’s equation
\[ U = \frac{k}{n} \left( R_h \right)^{2/3} (S)^{1/2} \]

Assume that stream width is much greater than stream depth
\[ U = \frac{k}{n} \left( d_{stream} \right)^{2/3} (S)^{1/2} \]

Rearrange to solve for stream depth
\[ d_{stream} = \left( \frac{U \cdot n}{k \cdot (S)^{1/2}} \right)^{3/2} \]
\[ d_{stream} = \left( \frac{U n}{k} \right)^{3/2} S^{-3/4} \]

Use the following table to answer questions 6 and 7:

<table>
<thead>
<tr>
<th></th>
<th>Channel 1</th>
<th>Channel 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean channel velocity</td>
<td>12.0 m/s</td>
<td>12.0 m/s</td>
</tr>
<tr>
<td>average stream width</td>
<td>1.5 m</td>
<td>1.5 m</td>
</tr>
<tr>
<td>channel bottom</td>
<td>smooth sand</td>
<td>smooth sand</td>
</tr>
<tr>
<td>slope</td>
<td>0.0006 m/m</td>
<td>0.0016 m/m</td>
</tr>
</tbody>
</table>

Q7) What is mean stream depth for the channel with a lesser slope? (4 points)
\[ d_{stream} = \left( \frac{U n}{k(S)^{1/2}} \right)^{3/2} = \left( \frac{12.0 \cdot 0.025}{1.0 \text{ m}^{1/3} \text{ s}^{-1} \cdot (0.0006)^{1/2}} \right)^{3/2} = 42.9 \text{ m} \]

Q8) What is mean stream depth for the channel with a greater slope? (4 points)
\[ d_{stream} = \left( \frac{U n}{k(S)^{1/2}} \right)^{3/2} = \left( \frac{12.0 \cdot 0.025}{1.0 \text{ m}^{1/3} \text{ s}^{-1} \cdot (0.0016)^{1/2}} \right)^{3/2} = 20.5 \text{ m} \]

Note: These calculations gave pretty unrealistic results, because the assumption that the width is much greater than the depth was invalid. A much lower mean channel velocity (and removing the average stream width info) would have made the problem more realistic.

**Effect of stream depth**

We can rewrite Manning’s equation to calculate the discharge as
\[ Q_{stream} = AU \]
\[ = A \frac{k}{n} \left( R_h \right)^{2/3} (S)^{1/2} \]

Q9) Using the assumption that the channel width is much greater than the depth, rewrite the preceding equation so that it is in the form
\[ Q_{stream} = \text{(constant parameters)} \times (d_{stream})^b \]
where \( b \) is the exponent applied to the depth of the stream. (4 points)
\[ Q_{\text{stream}} = A U \]
\[ = A \frac{k}{n} (R_h)^{2/3} (S)^{1/2} \]
\[ = w d_{\text{stream}} \frac{k}{n} (d_{\text{stream}})^{2/3} (S)^{1/2} \]
\[ Q_{\text{stream}} = \left( w \frac{k}{n} (S)^{1/2} \right) (d_{\text{stream}})^{5/3} \]

Q10) Based on the equation derived in Q9, what effect will an increased depth have on the discharge of the stream? Graph the equation, with the depth of the stream on the x-axis and discharge on the y-axis. (4 points)

Discharge increases with the 5/3 power of average stream depth. This is assuming that the width is constant, but usually the width also increases as the depth increases.